2

Scientific Notation

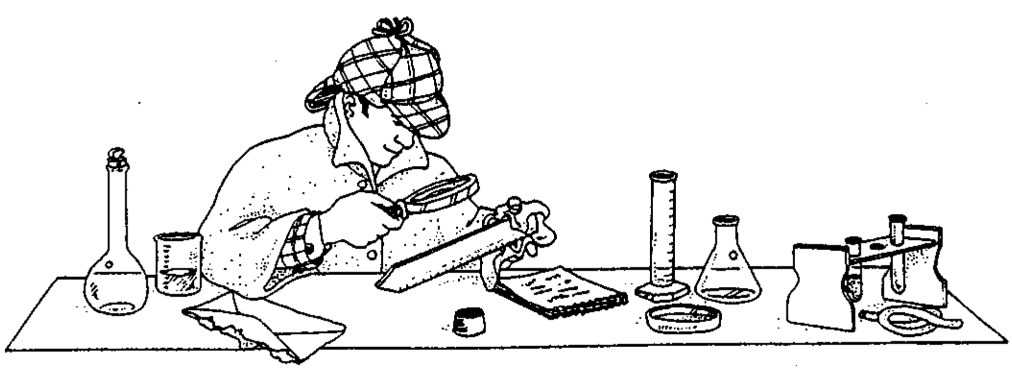
2.1 ANOTHER TRUE(?) STORY

Nobody in the Science Department could believe it. Professor Brillium was accused of stealing a secret formula—one for the synthesis of a molecule that might be able to prolong life. Professor Brillium was a gentle scholar known for his kind manner and absent-mindedness. He was not as absent-minded, perhaps, as the great Ampére, the French scientist after whom the unit for electric current is named. (That gentleman was once crossing a bridge when he saw an interesting rock. He picked up the rock and was looking at it when he remembered that he was on his way to keep an appointment. Taking his watch out of his pocket, he saw that he was quite late, hurriedly tucked the stone in his pocket, threw the watch into the water, and rushed on.) No, Brillium was absent-minded, but not that absent-minded.

Inspector Cloozouch, in charge of the investigation, noted at once that a ruler on Brillium's table, pointed at one end so that it could be used as a letter opener, was the main clue. It had Brillium's fingerprints on it also. The thief had apparently entered the adjoining laboratory next to that of Brillium's, opened a locked drawer with a master key, removed a sealed envelope containing the formula, opened the envelope with the pointed ruler, and removed the contents of the envelope. Then, probably hearing someone approaching, the intruder had tossed aside the envelope and disappeared. The only place the intruder could have gone was through the connecting door to Brillium's lab.

"It is clear that Brillium is the thief," said Cloozouch. This ruler was surely used to open the secret formula envelope. It has a serrated edge that matches the cut in the envelope.

Brillium claimed that he had never seen nor touched the ruler before.



"Arrest him," cried Cloozouch.

"No, no," said his colleagues and students. "Brillium is an honest, upright scientist even if he is a little absent-minded."

In desperation, the faculty and students called the famous detective, Herlock Shohmes, to help them. He appeared, and slowly and carefully examined the clues. Then he stared at Cloozouch for a long, silent moment. Finally, he said, "Cloozouch, if you knew the least thing about scientific notation and experimentation, you wouldn't make such ridiculous accusations."

"Tell me," said Shohmes to Brillium, "has anyone been here recently besides you?"

"Yes," said Brillium, "a student handed in a report."

"What did the student look like?"

"I didn't see him," said Brillium. "I just held up my hand and he put the report into it. I placed the report on the table without looking and went on working." Brillium pointed to the table but all that was there was the ruler. "Oh," said Brillium, "I guess he handed me the ruler, and *that* was what I put on the table."

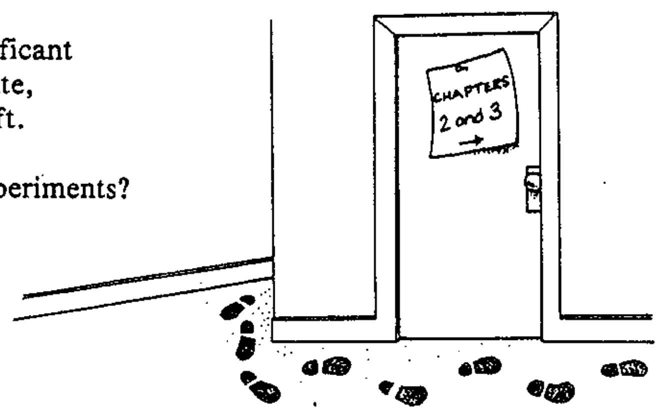
"How could Brillium be so absent-minded?" objected Cloozouch.

"I believe his story," said Shohmes, "because he could not have been working with that ruler now, or at any other time. Look at the data in his book dated the day of the theft and earlier. It says, for example, 1.243×10^{-4} m, 2.062×10^{-4} m, and 7.41×10^{-5} m. That is conclusive proof that the professor was not using this ruler at the time of the theft, and that probably he did not use it in the laboratory at all.

"Hooray for science!" cried everyone. "Hurray for scientific notation and significant figures," and they all went off to celebrate, leaving Cloozouch alone to solve the theft.

How did Herlock Shohmes know that Brillium did not use that ruler for his experiments?

To use the clues in the story, complete Chapters 2 and 3.



2.2 SCIENTIFIC NOTATION

The system of scientific notation has certain practical advantages that will be shown later in this chapter and in the chapter on significant figures.

In this system, *numbers are written in the form* $A \cdot 10^x$ *where A is a number with the decimal point placed after the first digit. The first digit must be a number from one to nine;* it does *not* include zero.

Below is a table of numbers showing the starting number and its form in scientific notation.

Starting Number	*Scientific Notation*
34 600	3.46×10^4
3460	3.46×10^3
3.46	3.46 or 3.46×10^0
0.0346	3.46×10^{-2}
0.000 346	3.46×10^{-4}

As you can see, the *coefficients of all numbers expressed in scientific notation are less than ten and equal to or bigger than one.*

Problem

1. Which of the following numbers are expressed in scientific notation?

a.	2.90×10^2	f.	0.80×10^3
b.	9.9×10^5	g.	9.6×10^0
c.	84.2×10^{-6}	h.	9.600×10^{-21}
d.	7.08×10^{-18}	i.	9600
e.	6.2×10^1		

2.3 HOW TO CONVERT AN ORDINARY NUMBER TO SCIENTIFIC NOTATION

Let us again examine the table of numbers showing the starting number and its form in scientific notation.

Starting Number	*Scientific Notation*
34 600	3.46×10^4
3460	3.46×10^3
3.46	3.46
0.0346	3.46×10^{-2}
0.000 346	3.46×10^{-4}

Notice that, when the coefficient of the number in scientific notation is smaller than the starting number, the exponent is positive. This compensates by making the rest of the scientific number correspondingly bigger, as in converting 34 600 to 3.46×10^4. Correspondingly, when the coefficient is bigger than the starting number, the exponent is negative to reduce it back again, as in converting 0.0346 to 3.46×10^{-2}.

To convert a number to scientific form, it must be rewritten as follows:

1. **Move the decimal point so that only *one* non-zero digit appears to the left of the decimal point. (This makes the coefficient equal to or greater than one but less than ten.)**
2. **Multiply the coefficient by the power of ten needed to keep the expression equal to the starting number.**

Example No. 1 Convert 46 000 to scientific notation.

First, the decimal point is moved four places to the left, 4.6 0 0 0

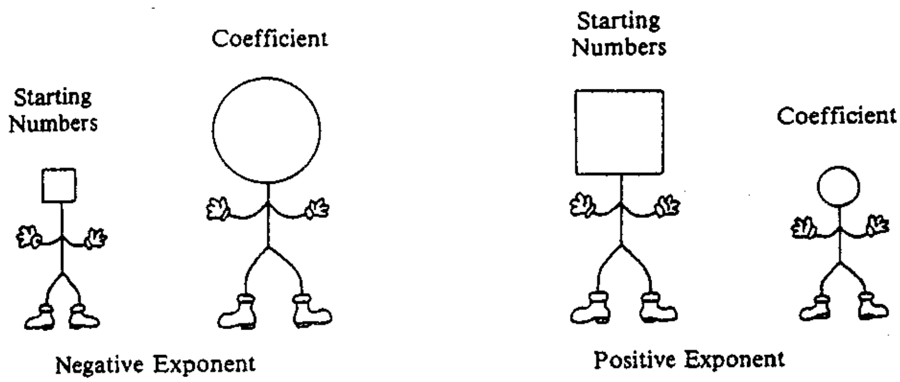
To maintain the number at its initial value, it must be multiplied by 10^4, so it is written as 4.6×10^4.

Example No. 2 Convert 0.004 60 to scientific notation.

0 0 0 4.60 The decimal point is moved three places to the right.

To maintain the number at its initial value, it must be multiplied by 10^{-3}, so it is written as 4.6×10^{-3}.

To shortcut the procedure, just count the number of places the decimal point is moved; this is the same number as the exponent. If the coefficient is larger than the starting number, the exponent is negative (to make the exponential part correspondingly smaller). If the coefficient is smaller than the starting number, the exponent is positive (so that it compensates by increasing it).



2. Convert the following to scientific notation.
   a. 356
   b. 21
   c. 45.627
   d. 9300
   e. 93 000 000
   f. 4.82
   g. 993.05
   h. 0.937
   i. 0.020
   j. 0.000 625
   k. 0.000 000 000 000 602

> **To convert a number back from scientific notation to its ordinary form, you need only follow the instructions given by the exponent. If the exponential part of the number is 10^x, move the decimal point x places to the right. If it is 10^{-x}, move it x places to the left. For example:**
>
> **$3.6 \times 10^3 = 3600$**
>
> **$3.6 \times 10^{-3} = 0.0036$**

3. For the following problems, convert the number from scientific notation to the non-exponential form.
   a. 6.3×10^3
   b. 6.3×10^{-3}
   c. 9.020×10^2 (include the zeroes)
   d. 9.020×10^{-2} (include the zeroes)
   e. 7.22×10^{-8}
   f. 1.00×10^4

2.4 COMPARISONS

In Section 1.10, numbers written in exponential form with a coefficient of one or two were compared to each other. Now, you can readily make comparisons of any two exponential numbers. As a result, you can also *make comparisons of any very large and/or very small* *numbers by first converting them to scientific notation and then making the comparison.* For example, to compare 186 000 centimeters to 0.000 062 2 centimeters, first convert both numbers to scientific notation. They become 1.86×10^5 and 6.22×10^{-5}. Using the methods learned so far,

$$\frac{1.86 \times 10^5\,\text{cm}}{6.22 \times 10^{-5}\,\text{cm}} = 0.294 \times 10^{10}\,\text{cm} = 2.94 \times 10^9\,\text{cm}.$$

Hence, the numerator is 2.94 billion times as big as the denominator, or 2 940 000 000 times as big. For the following, place the first number in the numerator for the comparisons.

4. Compare 2.4×10^8 to 5.6×10^4.
5. Compare the speed of light at 3.00×10^8 m/sec to the U.S. legal auto speed limit of 24.5 m/sec.
6. Compare 3×10^{-6} to 9×10^{-9}.
7. Compare 9.06×10^{-9} to 4.03×10^{-6}.
8. Compare 0.000 006 54 to 0.000 000 000 009 82.
9. Compare 10 462 000 000 000 to 3 295 000.

As you can see, it is especially easy to compare numbers when they are expressed in scientific notation. Scientific notation also makes estimated comparisons simple to calculate. To make estimations, all that is needed, essentially, is to round off the coefficients of the numbers, expressed in scientific notation, to the nearest integer.

Example No. 3 Compare 2.4062×10^9 to 1.1070×10^6 by estimation.

Rounding these off, we have $\dfrac{2 \times 10^9}{10^6} = 2 \times 10^3$ or 2000 times as much.

Example No. 4 Compare 1.624×10^{-3} to 1.525×10^{-6} by estimation.

Rounding off: $\dfrac{2 \times 10^{-3}}{2 \times 10^{-6}} = 10^3$ or 1000 times as much.

10. Compare by estimation: (place the first number in the numerator).
    a. 9.625×10^8 to 2.061×10^6
    b. 3.90×10^4 to 1.86×10^8
    c. 1.76×10^{-6} to 1.09×10^{-5}
    d. 7.8×10^{-23} to 3.6×10^{-25}

Later in this book, in Chapter Four, the technique of estimation is further examined.

In this chapter, you have studied how to express numbers in scientific notation. The exponential system makes it easy to compare numbers. It also makes it easy to estimate large and small numbers and to compare the estimations. The next chapter will show a very important way to use scientific notation.