# 3
# Significant Figures

## 3.1 DAMOS AT THE FAIR

Once upon a time, a great fair was held near a big village. The teachers at a nearby school were allowed to bring their students for the afternoon. Each student had an allowance of five krunks to spend. One student, Damos, announced that he was going to use his money to its best advantage. Everyone laughed at him. "Your best advantage," they said, "is to fill your stomach with good food. That's all you'll be able to take back with you." Damos only smiled.

Presently, they passed a stand where a man guessed your weight for one krunk. If the man was wrong, you received a large red apple, but if he was right, he kept the krunk and the apple. His sign said, "I guess your exact weight or you get a red apple with your money back."

Damos gave the man a krunk. The man looked him up and down, patted his shoulders and back, and then said, "One hundred fifteen pounds." Damos stepped on the scale. Indeed, it did read just a little over one hundred fifteen pounds. Everyone was sorry for Damos, but he just said, "I'll take my apple and my krunk now." "What do you mean?" exclaimed the man. "I guessed your exact weight!" "Oh, no, you didn't," responded Damos. "Your scale does not read exactly one hundred fifteen pounds."

The man stared at Damos. The students stared at the man. The whole crowd stared at the man. "He's right," they exclaimed, "he's right! Your sign says 'exact' weight. Give him his prize."

Slowly, the man turned and took a large red apple from a pile in the back. Then, he gave Damos the apple and one krunk. He was reaching up to take his sign down when Damos and his friends walked away.

"There's no way that he can ever guess my exact weight," said Damos. "He's not the only one at this fair who doesn't know the meaning of an exact measurement. I will earn a full stomach and keep my money while I teach some others what it means."

What is an exact measurement? Read further to find out.

## 3.2 MEASURED NUMBERS ARE DIFFERENT

Unlike the world of pure mathematics with its abstract numbers, such as exactly two or exactly three, scientists often measure and count with the aid of instruments that are limited as to how precise they can be. No matter how the instruments and methods are refined, there is no way ever to get the ultimate exact value of any quantity measured with a tool.

In this chapter, we are going to further explore the world of quantities, a world where there are many quantities that can never be exactly known, a world where fundamental scientific values such as the Avogadro Number have changed over the years even though they are constants. The Avogadro Number, which specifies the number of particles in a mole of substance, changes as methods of measurement improve. It used to be $6.02 \times 10^{23}$. Then, it became $6.022 \times 10^{23}$. After several more improvements in measurement, it became $6.022\ 045 \times 10^{23}$.

Now, it is $6.022\ 136\ 7 \times 10^{23}$. Each new number results from being able to measure it more precisely. Even now, we are still unsure of the "7" at the end of the present measurement, and don't know any of the sixteen digits following it.

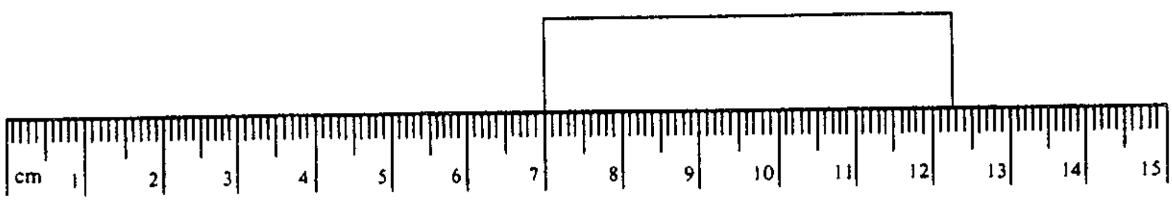
## 3.3 WHAT MAKES A MEASUREMENT INEXACT?

Before we look at the system of significant figures, let us examine a little more closely what kinds of measurements can be exact or inexact.

The *only* measurements that can be made exactly are those that measure separate items that can be counted one by one. For example, if there are twenty-one students to a classroom, or eleven players on a football team, or one hundred sheets of paper to a pack, the numbers are no more and no less than 21, or 11, or 100. Such a measurement is exact only because there can be a *one-by-one count.* All other types of measurement are inexact.

There are a variety of causes of inexactness, but there is one that is built into every measurement except counting item by item. It is built in because it requires the use of a measuring tool or instrument. Let's see why this is so.

Suppose you wish to measure the length of a cardboard box. Placed on a ruler, it looks like this.




One side of the box is set at 7 cm. What is the ruler reading for the other side? It is somewhere between 12.2 and 12.3 cm; it is definitely at least 12.2. We will have to estimate the last digit. One person may read it as 12.24 while another may see it as 12.25 or 12.26; the last digit (4, 5, or 6) is inexact.

Every such measured quantity (except one-by-one counting) has a last digit in it that must be estimated. Here is another example.


What is the reading on the scale where the arrow crosses it? Which digit did you have to estimate?

We saw it as 3.24. The 4 was estimated.

*The number part of all inexact measured scientific quantities consists of a series of digits where the last one to the right is estimated.* The total digits in the number depends on the precision of the measuring tool.

Suppose you wanted to measure the width of a sheet of paper. You could measure it to 21 cm or $21\frac{1}{2}$ cm or 21.52 cm or, with the aid of a magnifying glass, to 21.526 cm. Instead of using a ruler, you might use a more precise instrument that would enable you to measure the width to 21.5257 cm. Using an even more precise instrument, you might extend this further. Perhaps, with the aid of a laser, the width could be found to be 21.52576811 cm. Yet, no matter how elaborate a measuring device you used, you could never get the measurement to be finally exact. Each more sensitive instrument would only allow you to add on another digit. That other digit might be very important—it might keep a rocket from missing the moon; but it would not make the number exact, only more precise.

Respond to the following.

1. Which of the quantities below are exact?
   a. John counted out ten dollar bills.
   b. There are 12 eggs to the dozen.
   c. The baker measured 2 pounds of butter and a quart of flour.
   d. Each box of chocolate contains 16 candies.
   e. The room is 13 feet by 18 feet.
   f. Please let me have 5 meters of that ribbon.
   g. The temperature is 23.4°C.
2. For each of the following quantities, recorded as part of certain scientific experiments, identify the digit that is uncertain (estimated).
   a. 2.063 cm
   b. 0.0089 kg
   c. 7.62 g
   d. 8.9 km
   e. 0.8 m
   f. 7.000 lbs.

## 3.4 SIGNIFICANT FIGURES

**When a quantity is measured scientifically and recorded, all the digits in the measured number are written down including the last one to the right that has to be estimated. The digits in such a number are said to be significant.**

If a quantity is reported as 6.045 cm, the "5" is doubtful or estimated, and the number has four significant figures (or four significant digits).

3. For the following pairs of quantities, reported using significant figures, which member of each pair has more significant figures?
   a. 6.321 cm and 6.3214 cm
   b. 1.234 g and 0.981 62 g
   c. 1.234 g and 0.9816 g
   d. 542 kg and 2.1 kg
   e. 543°C and 24.2°C
   f. 1282 g and 128 g

   Remember: the zero in front of the decimal point is only a marker to make sure the decimal point shows. *It doesn't have to be there, so don't count it as a digit.*
4. For each of the quantities in the preceding problem 3, state the estimated digit.

5. For the following measured quantities, state which "possible range" must be wrong, considering that the quantities were reported using the system of significant digits. Some sample answers are provided; you do the rest.

| | *Quantity reported* | *Possible range* | *Right or wrong, and why* |
|---|---|---|---|
| a. | 0.9616 liters | 0.9617 to 0.9615 liters | Right, because it is the last 6 that is estimated. |
| b. | 545 kg | 530 to 550 kg | Wrong, because only the 5 is estimated, not the 4. |
| c. | 3.84 m | 3.82 to 3.86 m | Right, because the 4 is estimated. |
| d. | 21.30 cm | 21.29 to 21.31 cm | Right, because the zero is estimated. The estimate spills over onto the preceding digit. |
| e. | 9652 g | 9650 to 9655 g | ________________ |
| f. | $6.3 \times 10^{-4}$ mm | $6.1 \times 10^{-4}$ to $6.5 \times 10^{-4}$ mm | ________________ |
| g. | 9.36 pascals | 9.26 to 9.46 pascals | ________________ |
| h. | 82.71 mL | 82.69 to 82.73 mL | ________________ |

## 3.5 ROUNDING NUMBERS

In the next sections, the rules for carrying out calculations using significant figures will be examined. Because calculations using significant figures require rounding off, it is important to look at and to practice the rules for rounding off.

When a number is to be rounded off, the *first digit to be dropped (counting from left to right) tells us what to do.*

> **RULE 1: If the digit in the first place to be dropped is less than 5 (is 4, 3, 2, 1, or 0), simply discard all the unwanted digits.**

*Example:* Round off 6.352 46 to two decimal places (three significant figures).

digits to be dropped

6.35246

preceding digit    first digit to be dropped

Since the first digit to be dropped, 2, is less than 5, all of the unwanted digits are dropped. The answer is 6.35.

*Example:* Round off $3.8519 \times 10^5$ to three significant figures.

digits to be dropped

$3.85\overbrace{19} \times 10^5$

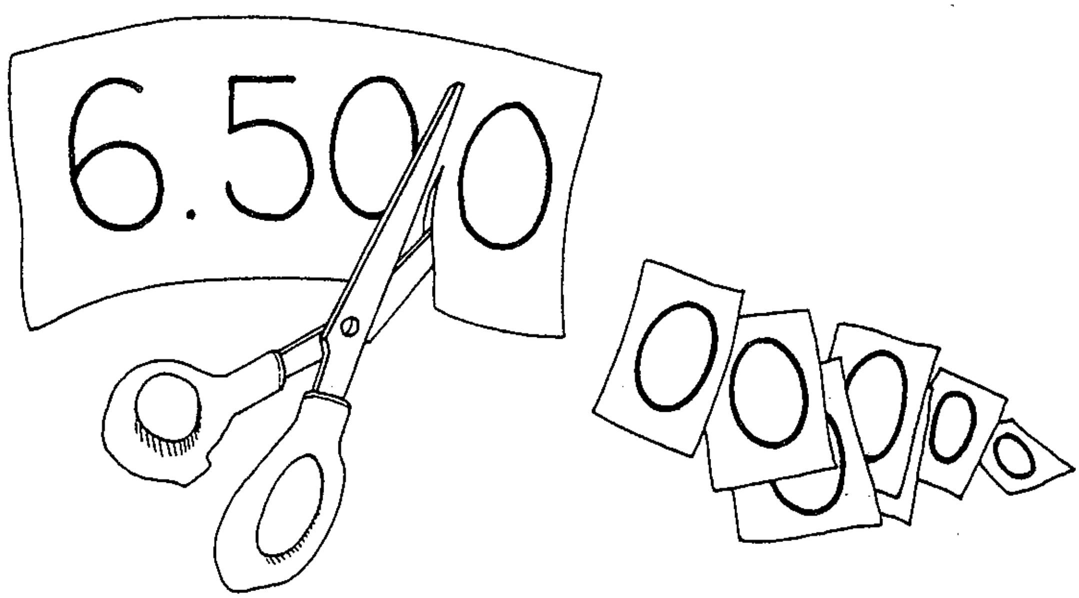
preceding digit — first digit to be dropped

Since the first digit to be dropped, 1, is less than 5, all the unwanted digits are dropped. The answer is $3.85 \times 10^5$.

| **RULE 2:** | **If the digits to be dropped are greater than 5, or greater than 5 followed by zeroes (greater than 5, 50, 500 or 5000...), increase the preceding digit by one.** |
|---|---|

When rounding each of the following to three significant figures,

3.7865 becomes 3.79
2.9651 becomes 2.97
83.4501 becomes 83.5
0.96062 becomes 0.961
$2.167 \times 10^{-5}$ becomes $2.17 \times 10^{-5}$



**RULE 3: Whenever the digit(s) to be dropped are either exactly 5, or 5 followed by zeroes, a way has been developed to increase the digit to the left about half the time and to leave it unchanged the remainder of the time. The system is simply that the preceding digit becomes the nearest even digit. That means that 1 is added to the preceding digit if it is odd, but nothing is added if the preceding digit is even.**

To 2 significant figures, 6.25 becomes 6.2.
To 2 significant figures, 6.35 becomes 6.4.
To 2 significant figures, 6.3500 becomes 6.4.
To 1 significant figure, 3.5 becomes 4.
To 1 significant figure, 2.5 becomes 2.

Not all teachers and textbooks use Rule 3. Your teacher will advise you as to whether it is to be used in your course. If Rule 3 is not used, then Rule 2 is extended to include every case where the first digit to be dropped is 5. That is, Rule 2 becomes: If the first digit to be dropped is 5 or greater than 5 (5, 6, 7, 8, or 9), increase the preceding digit by 1.

*Problems*

6. Round the following atomic masses first to three significant digits and then to four significant digits:

| | | | | | |
|---|---|---|---|---|---|
| aluminum | 26.98514 | fluorine | 18.998403 | mercury | 200.59 |
| argon | 39.948 | gold | 196.9665 | nitrogen | 14.0067 |
| carbon | 12.011 | helium | 4.00260 | oxygen | 15.9994 |
| chlorine | 35.453 | hydrogen | 1.0079 | silicon | 28.0855 |
| copper | 63.546 | iron | 55.847 | uranium | 238.029 |

7. Fill in the blanks in the chart below.

| Number | Number rounded off to: Four significant digits | Three significant digits | Two significant digits | One significant digit |
|---|---|---|---|---|
| 96.831 | | | | |
| 37.0000 | | | | |
| 0.9531 | | | | |
| 0.935 67 | | | | |
| 0.935 00 | | | | |
| 0.945 00 | | | | |
| 32.50 | | | | |
| 1 064 397 | | | | |

Now, we can move on to the study of calculations using significant figures.

## 3.6 ADDITION AND SUBTRACTION WITH SIGNIFICANT FIGURES

The rule for addition and subtraction using significant figures is different from that for multiplication and division. In the rule for addition and subtraction, the number of digits located to the right of the decimal point controls the number of significant figures in the answer.

> **In an addition or subtraction using significant figures, the answer can have no more digits to the right of the decimal point than are in the number that has the least number of digits to the right of the decimal point.**

This is illustrated by the following:

*Example No. 1* Find the sum:

| | |
|---|---|
| 3.2 | mL |
| 54.124 | mL |
| 278.03 | mL |

Since the least number of digits to the right of the decimal point in these numbers is one, the answer may have only one digit to the right of the decimal point. Round off each number to one decimal place.

| | |
|---|---|
| Rounded off: | 3.2 mL |
| | 54.1 mL |
| | 278.0 mL |
| The sum is: | 335.3 mL |

*Example No. 2* Find the difference:

| | |
|---|---|
| 54.6172 | kg |
| −3.306 | kg |

The smallest number of digits to the right of the decimal point is three, so the answer may have only three digits after the decimal point. Round off each number to three digits after the decimal point.

| | |
|---|---|
| Rounded off: | 54.617 kg |
| | −3.306 kg |
| Difference is: | 51.311 kg |

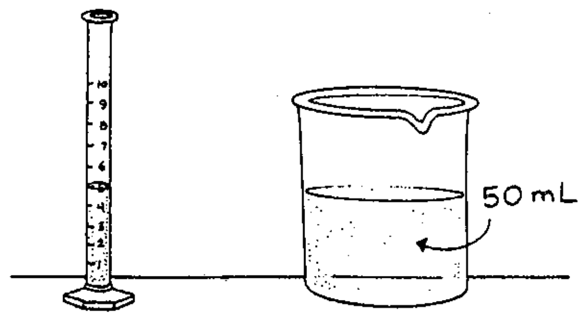
Below are other examples of additions and subtractions with answers.

| | | |
|---|---|---|
| 23.6 kg | 6.00 g | 2 cm |
| −16.1218 kg | 32.122 g | 5612.0493 cm |
| 7.5 kg | 0.0211 g | 13.1 cm |
| | 5.6231 g | 244.345 cm |
| | 43.76 g | 5871 cm |

To help understand why this is the rule for addition and subtraction of significant figures, here is an experiment for you to try. Obtain a 10 mL graduated cylinder and a 100 mL beaker. Fill the beaker with sink water about half full; do the same for the cylinder. Now, read the amounts of water in each and write down the observation. In each case, use the proper number of signifcant figures; that is, record the number to the first estimated digit. Next pour the contents of the graduated cylinder into the beaker and record the new volume of water.

The graduated cylinder can be read to about 0.1 mL while the beaker can only be read to within 5 to 10 mL. Thus, the following data might have been obtained.

| | |
|---|---|
| Volume in cylinder | 5.12 mL |
| Volume in beaker | 50 mL |
| Total volume | 55 mL |




graduated cylinder    beaker

Your data will look similar to the preceding; in fact, the total volume may even appear to be 50 mL, since the beaker can only be read to within 5 to 10 mL. The last digit is estimated or doubtful in each case. The initial volume in the cylinder might have been read as 5.11, 5.12, or 5.13 mL, while the initial volume in the beaker may have been read as 45, 50, or 55 mL. There is no need at all to be careful about measuring the volume in the graduated cylinder before adding it to the beaker. The fact that the beaker can only be read to whole numbers controls the *sum* of the volumes. Hence, in this case, the estimated or doubtful digit in the sum is the one before the decimal point, and there are no significant digits after the decimal point.

The sum or difference can only be as precise as the least precise quantity.

The examples given earlier in this section all involved rounding off by dropping the unwanted digits. Below are some examples that also include rounding up to the next higher digit.

| | | |
|---|---|---|
| 6.2907 | rounds off to: | 6.3 |
| 33.5 | | 33.5 |
| 4.656 | | 4.7 |
| 0.052 | | 0.1 |
| 4.45 | | 4.4 |
| 0.028 | | 0.0 |
| | Sum = | 49.0 |

| | | |
|---|---|---|
| 36.159 | rounds off to: | 36.16 |
| −4.50 | | 4.50 |
| | Difference = | 31.66 |

Sometimes, it is easier to do the computation first and round off at the end. If rounded off afterwards, the addition above would have totaled 48.9767, which should be rounded off to 49.0. Occasionally, two such sums may differ in the last digit because of the difference in rounding off.

Solve the following problems. The numbers are all measured with the system of significant figures.

8. 7.623 + 85.0 + 9.815

9. 230.72 + 0.00861 + 9.7250

10. 10.96 − 5.5

11. 10.96 − 5.555

12. 9.0731 + 0.00078

13. The sum of
 4.7539
 61.268
 0.0098
 15.45
 9.682

## 3.7 PLACEHOLDERS

The system of using zeroes as placeholders can result in some unclear meanings in the understanding of significant figures; special precautions must be taken for such numbers. Two examples follow to show how this can happen.

Suppose a large beaker is half-filled with water. If the beaker holds about 500 mL in all, it now holds about 250 mL. How many significant figures are there in this quantity?

The answer is that there are two, since the beaker may hold 250, 240 or 260 mL, or thereabouts. The second digit is estimated and the zero is merely a placeholder.

To show that the zero is only a placeholder, let us convert the measurement to liters.

$$250\,\text{mL} \times \frac{\text{one liter}}{1000\,\text{mL}} = 0.25 \text{ liters}$$

Since there are only two significant figures in the measurement, only two are needed for the quantity when expressed in liters. Notice that *the zero to the left of the decimal is not significant;* it can be omitted.

In another case, the quantity 50.0 kg is converted to grams. Both zeroes in this quantity are significant since the measurement was evidently taken to $\frac{1}{10}$ kg. In grams, however, the quantity now appears as 50 000 grams. Unless the reader has special information, there is no way to tell that only two of these zeroes are significant.

The use of scientific notation solves this problem neatly. The quantity, 50.0 kg, is recorded by the system of scientific notation as $5.00 \times 10^1$ kg or as $5.00 \times 10^4$ g. The significant digits are shown by the coefficient; *only significant figures may appear in the coefficient.*

Thus, the following all show quantities with four significant digits:

$4.162 \times 10^3$, $6.250 \times 10^{-8}$, $9.000 \times 10^{-4}$, $9.000 \times 10^4$, $6.250 \times 10^8$.

For these last two numbers, notice that you cannot tell from the ordinary numbers, 90 000 or 625 000 000 how many significant figures each has.

Notice, too, that the number $6.250 \times 10^{-8}$ looks like .000 000 062 50 in its ordinary form. All seven zeroes to the left of the 6 are placeholders; they are not significant; the zero to the right is significant. It is significant because according to the coefficient, it must have been the estimated digit in the measurement. This number might also be written as

$6.250 \times 10^{-8}$ kg or 0.000 000 062 50 kg
$6.250 \times 10^{-5}$ g or 0.000 062 50 g
$6.250 \times 10^{-2}$ mg or 0.062 50 mg
$6.250 \times 10^{1}$ micrograms

**The location of the decimal point does not affect the number of significant figures in a measurement.**

To repeat, all the zeroes at the left serve only as placeholders.
The system of scientific notation shows unmistakably which measured numbers are significant.

*Problem*

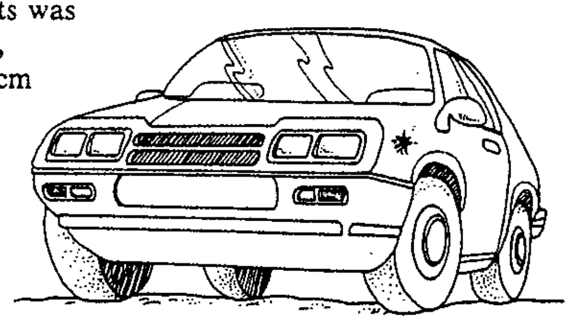
14. How many significant figures are in each of the following measurements, recorded using the system of significant figures?

| | | | |
|---|---|---|---|
| a. | 634 g | g. | 0.01 mol |
| b. | $6.300 \times 10^{3}$ g | h. | 0.0042 joule |
| c. | $6.3 \times 10^{2}$ kg | i. | $6.430 \times 10^{12}$ hertz |
| d. | 21.00 cm | j. | $6.40 \times 10^{-12}$ eV |
| e. | 0.1 pascal | k. | 0.002 005 mm |
| f. | 205 cm | l. | $2.005 \times 10^{-3}$ mm |

## 3.8 MULTIPLICATION AND DIVISION WITH SIGNIFICANT FIGURES

The rectangular area of a repair patch needed for the fiberglass body of a car is measured. It turns out to be 1.23 cm by 2.54 cm. To get the area, the two measurements are multiplied. On a calculator, the answer comes out to 3.1242 cm². However, the answer is correctly recorded to only three significant figures or 3.12 cm². Recording any more numbers is a waste of time and can even, as will be shown later, be expensive in some cases.

You may well ask why only three significant figures are used in the product. To explain, recall that the last digit in each of the measurements was estimated. Suppose that instead of reading 1.23 cm, the observer saw it as 1.24 cm or 1.22 cm; the 2.54 cm might have been seen as 2.53 or 2.55 cm. Let us calculate the highest and lowest results that might have been obtained for the area from such a measurement.



Highest: $1.24\,\text{cm} \times 2.55\,\text{cm} = 3.1620\,\text{cm}^2$
Lowest: $1.22\,\text{cm} \times 2.53\,\text{cm} = 3.0866\,\text{cm}^2$

We see that the area is between 3.09 and 3.16 cm², or about 3.12 cm². The last digit is uncertain. Thus, this number has only three significant figures, the same number as in each of the numbers that were multiplied together.

*The product of two or more numbers is as inexact as is the least exact of the numbers.*

A similar situation exists for division. This leads to the following rule:

> **Retain in the product or quotient the same number of significant digits as in the quantity with the least number of significant digits.**

If, for example, you divide 3.0 g by 13 mL to calculate the density of a certain sample, and your calculator shows 0.230 769 2, that is not the correct answer in g/mL. Such an answer implies that you have measured the mass of the sample to one ten-millionth (0.000 000 1) of a gram and the volume to one-millionth of a mL. The correct answer is $2.3 \times 10^{-1}$ g/mL or 0.23 g/mL; the former is preferable.

*If any of the numbers in the calculation have more significant digits than another, round each off to match the least number of significant figures in any of the numbers before starting the calculation.*

Solve the following problems; use scientific notation to write each answer to the correct number of significant digits.

*Problems*

15. Density equals mass divided by volume. Calculate the density of a chunk of iron that occupies 14.3 mL and has a mass of 112.398 g.
16. Given that the product of A and X is $2.634 \times 10^{-6}$ g²/mL², if A equals $2.00 \times 10^{-4}$ g/mL, what is X?
17. The solubility of sodium chloride in water at 0°C is 35.7 g per 100 mL. How much will dissolve in 1500 mL?

In problems 18 to 23, the measurement labels have been omitted from the calculations; assume that each of the numbers was recorded using the system of significant figures.

18. $\dfrac{(3.2 \times 10^{3}) \times (4.21 \times 10^{2})}{6.28 \times 10^{-8}}$
19. $2.695 \times 33.20 \times 1.5611$
20. $\dfrac{8.032}{0.0591}$
21. $(72.21 \times 6.42) + 7.050$
22. $\dfrac{6\ 000\ 260 \times 0.005\ 201}{5.206}$
23. $2.500 \times 10^{6} \times 3.92 \times 10^{-3}$
24. A room is 12 ft. by 12 ft.; these quantities were recorded using significant figures. What is the area of the room, using significant figures and scientific notation?

25. The pressure $p$, of a container of helium was calculated from the formula $p = \frac{nRT}{V}$

*where* $n = 1.000$ mol

$R = 0.0821 \frac{\text{liter} \cdot \text{atmosphere}}{\text{mol} \cdot \text{K}}$

$T = 293$ K

$V = 5.216$ liters.

Determine $p$; don't forget the units.

26. How many molecules are in 1.000 liter of oxygen at zero degrees Celsius, and one atmosphere pressure, given that there are $6.02 \times 10^{23}$ molecules of oxygen in 22.4 liters at the same temperature and pressure?

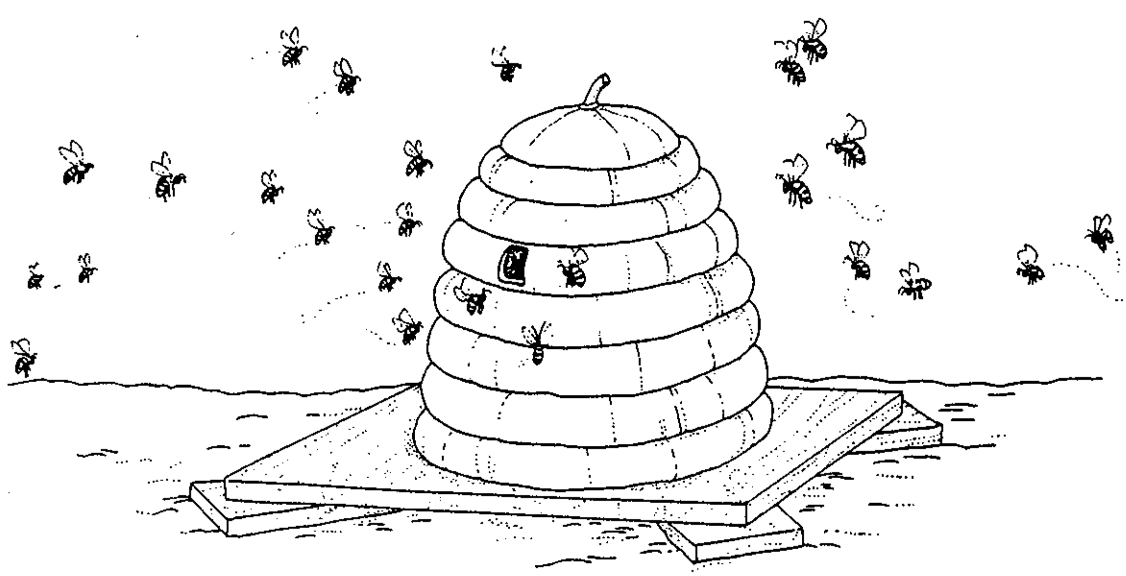
## 3.9 EXACT QUANTITIES AND SIGNIFICANT FIGURES

The system of significant figures refers only to inexact quantities. *When both exact and inexact numbers are used together in calculations, assume that the exact number has as many significant figures as needed.* To do this, let us first examine the different kinds of exact numbers.

### A. Exact Integers

Suppose that you are asked to calculate the cost of three books, given that each book costs $6.73. Note that the "three books" and "each book" are exact quantities. For example, the 3 may be written as 3, 3.0, 3.00, 3.000. . . , etc. It has an infinite number of significant zeroes after the decimal point that may be called up when needed in computations using significant figures.

The rule of calling up as many significant zeroes after the decimal point as are needed holds for all exact integers.



The numbers may be exact because they are given as exact, or because they have been counted item by item. Ordinarily, you can recognize from your own experience when a quantity is exact. Most money transactions are exact; in the above example, each book was exactly \$6.73, not about six dollars. Although counting item by item is usually exact, there are even exceptions to this, as when counting by eye the number of bees buzzing around a hive or the number of people running around in a football stadium after a championship game.

Although an exact quantity such as ten pennies may, if desired, be written as 10.0 or 10.00 or 10.000. . .pennies, it is not necessary to actually write down the zeroes after the decimal point if the number is known to be exact.

For example, if there are 2.50 kg of rice in 1 bag, how many are there in 25 bags? We know, from experience, that the 1 bag and the 25 bags are exact quantities, since they can be counted one by one. Hence

$$\frac{2.50\ \text{kg}}{1\ \text{bag}} \times 25\ \text{bags} = 625\ \text{kg}.$$

The answer has three significant figures in it, because the measured quantity of 2.50 kg has three significant figures. Had each of the bags held 2.500 kg, then

$$\frac{2.500\ \text{kg}}{1\ \text{bag}} \times 25\ \text{bags} = 625.0\ \text{kg}.$$

Some authors of science books indicate exact numbers in the problems sections of the books by writing out the number instead of using numerical digits. In that way, the author does not have to make any assumptions about the reader's previous experience. For example, if a problem says "one meter" or "ten moles" or "five centimeters," all of these are exact and may be treated as exact in the calculations. Thus, the problem before might have been written, using significant figures, as

If there are 2.50 kg rice in one bag, how many are there in twenty-five bags?

In that case, the reader *must* treat the spelled-out quantities as exact.

*Problems*

27. From the "given" below, see if you can tell which of the following are exact.
    - a. 3 jars
    - b. 40 centimeters
    - c. forty centimeters
    - d. $6.02 \times 10^2$ beans
    - e. $6.02 \times 10^3$ beans
    - f. $6.02 \times 10^{-3}$ kg
    - g. 43 grains of wheat
    - h. 43 mL
28. Which of the following are very likely to be exact?
    - a. 36 people counted one by one
    - b. 12 inches measured with a ruler
    - c. the number of drops of water in the Atlantic Ocean.
29. True or false: Since science deals mainly with measured quantities, it follows that science deals mainly with inexact numbers.

**B. Defined Equalities**

There is a type of exact number often used in science calculations that arises out of defined equalities such as

one meter equals one thousand millimeters;
one kilogram equals one thousand grams;
twelve inches equals one foot.

These quantities are *defined* as being exactly equal to each other; hence, each equality connects exact quantities.*

$$\left.\begin{aligned} 1\,\text{m} &= 1000\,\text{mm} \\ 1\,\text{kg} &= 1000\,\text{g} \\ 12\,\text{in.} &= 1\,\text{ft.} \end{aligned}\right\}$$ In calculations using significant figures, each of these quantities may be treated as exact.

For example, convert $6.02 \times 10^2$ g to kilograms. The solution is

$$6.02 \times 10^2\,\text{g} \times \frac{1\,\text{kg}}{1000\,\text{g}} = 6.02 \times 10^{-1}\,\text{kg}$$

The number of significant figures in the answer depends only upon the number of significant figures in the measured mass.

In problems 30 to 34 below, carry out the conversions using scientific notation and the system of significant figures.

30. Convert 0.062 g to kilograms.
31. Convert $9.60 \times 10^{12}$ g to milligrams.
32. Convert $9.145 \times 10^2$ g to kilograms. How much is one-fourth of that?
33. Convert two-thirds of 3.06 L to milliliters.
34. Convert 0.00620 mL to liters.
35. If there are $2.5 \times 10^2$ BB shots in one container, how many are in 150 containers?
36. The mass of a hydrogen atom is $1.6734 \times 10^{-24}$ g. What is the mass of one thousand hydrogen atoms?
37. There are 172 Calories per pound of grapefruit juice. How many Calories are in exactly 1625 ounces of juice?

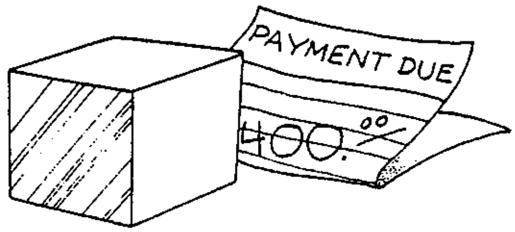
## 3.10 ONE MORE STORY**

There once was a graduate student who needed a cube of copper of a particular size for his research project. He knew that one of the other graduate students had had such a cube made for $30 at a nearby machine shop, so he sent the lab assistant over with a note saying, "Make

*This is different from *measured* equalities such as 1.00 kg = 2.20 lb., or 1.000 kg = 2.202 lb.

this cube one centimeter on each side." Eventually, the cube came back, and along with it came a bill for $400. The graduate student was shocked.

"How dare they think they can overcharge me?" he said, as he stormed over to the shop. Confronted by the irate student, the shop supervisor calmly said, "You wanted a one-centimeter cube. Because you wrote out the "one," this meant that you wanted it to the best precision we can machine, which is 0.001 cm. It takes quite a bit of careful grinding to get it to that precision. The first time we polished it, one side was too small, so we had to throw it out. The next time, there was a tiny bubble on the surface when it was almost to size, so we had to throw that out, too. It took four tries in all. That's why it was so expensive."




"Why did you do it for my friend for only $30?" demanded the student.

"Oh," replied the supervisor, "she asked to have it machined to 1.0 cm × 1.0 cm × 1.0 cm."

The use of significant figures can help save money; misuse can be expensive.

**The original source of this story is unknown. It was heard at a meeting held many years ago.